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FINAL REPORT.

Analytical Investigation of Two-Phase MHD Duct Flows.

(as requested by ONR officials, the research title was later changes to:

The Impinging Jet of Two-Phase Flows).

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ABSTRACT

We have completed a numerical scheme for approximating the free

boundary of a two dimensional inviscid incompressible jet impinging

perpendicularly upon a solid wall.   A Gould plot shows good agreement

with an exact analytical method described by Birkhoff and Zarantonello

1957.   Our method has the advantage over the analytic procedure that it

allows for more realistic physical hypotheses.   An example of a situation

to which our method could be extended is the axisymmetric jet, for which

no exact solution is know.   It should be emphasized, however, that our

algorithm is an approximating scheme; questions as to the speed of convergence

and relative merits over the finite elements scheme used by Sarpkaya and

Hiriart as reported by Gallagher and Oden 1975 in connection with the axi-

symmetric jet remain to be investigated.

Our work should be viewed as a first step towards developing a

predictive model for the velocity and pressure distributions in the

impinging jet of a two-phase medium.   The complexities of this process

require alternative approaches to those followed in the past.   This search

for new directions has been our primary objective.

A NUMERICAL SCHEME FOR THE IMPINGING JET

An algorithm is developed for predicting the free boundary of a 2-dimensional jet which impinges perpendicularly to a solid wall. The flow is assumed inviscid and incompressible.

The coordinate axes are chosen as in Fig. 1. The velocity components are denoted $u(x,y)$ and $v(x,y)$. The velocity at the nozzle exit is assumed of constant magnitude u_o. The half-width of the nozzle is y_o, and the distance from nozzle exit to wall is one.
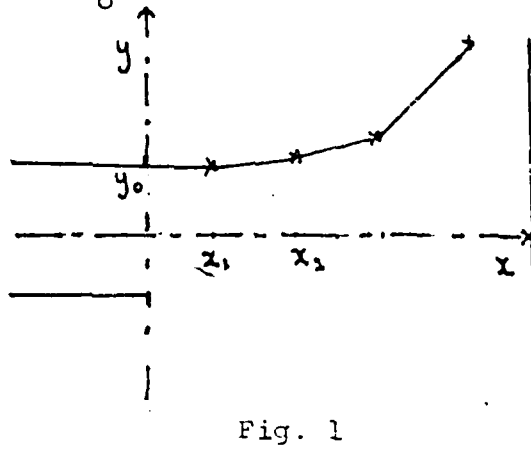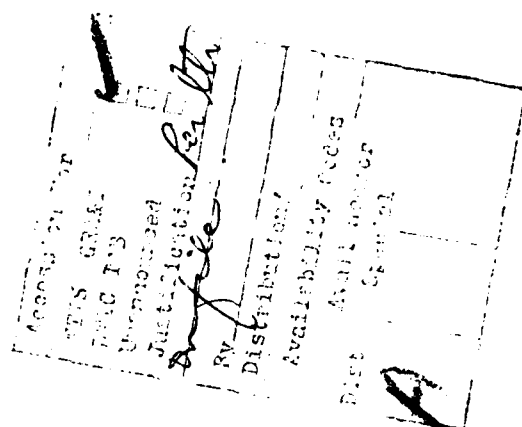

Fig. 1

Let $\psi(x,y)$ be the streamfunction. By adding a constant, if necessary, we can assume that ψ takes the constant value zero on the jet axis (which is a streamline). We then have:

$$(1) \qquad \psi(x,y) = \int_0^y u(x,y)\,dy$$

$$(2) \qquad \frac{\partial}{\partial x}\psi(x,y) = -v(x,y)$$

[The second term in the first equation represents the flux across a vertical line segment limited between (x,y) and $(x,0)$; we use the known fact that the flux across a curve is the difference in values

of the streamfunction at the endpoints.]

Algorithm. It suffices to determine the portion of the free boundary that lies above the jet axis.

Choose a positive integer n. Set $dx = \frac{1}{n}$, and define $x_1 = dx$, $x_2 = 2dx$, etc. Set $x_0 = 0$. Suppose the point (x_m, y_m), for some m between 0 and n, has been found to be on the free boundary, and that $v(x_m, y_m)$ has been determined.

We first approximate $\psi(x, y_m)$ by a 2nd order polynomial, i.e.

$$(3) \qquad \psi(x, y_m) = A_m(x - x_m)^2 + B_m(x - x_m) + C_m$$

The equations (1) and (2) yield:

$$\psi(x_m, y_m) = u_0 y_0$$
$$\psi(1, ym) = 0$$
$$\psi'(x_m, y_m) = -v(x_m, y_m)$$

which allow us to determine the constants in (3)

$$C_m = u_0 y_0$$
$$B_m = -v(x_m, y_m)$$
$$A_m = -\frac{C_m + B_m(n-m)dx}{(n-m)^2 dx^2} \quad ,$$

Equations (2) and (3) yield:

(4) $\psi'(x_{m+1}, y_m) = 2A_m dx + B_m = -v(x_{m+1}, y_m)$

The slope of the free boundary at (x_m, y_m) will be:

(5) $$s_m = \frac{v(x_m, y_m)}{u(x_m, y_m)} \quad .$$

Bernoulli's equation on the free boundary gives us:

(6) $$u(x_m, y_m) = \sqrt{u_o^2 - v(x_m, y_m)^2} \quad .$$

Thus if we know $v(x_m, y_m)$, we can determine s_m, and approximate

(7) $$y_{m+1} = y_m + s_m dx$$

[The above consists in assuming that the free boundary is a line segment between (x_m, y_m) and (x_{m+1}, y_{m+1}).]

We make a final approximation, and use (4) to get

(8) $$v(x_{m+1}, y_{m+1}) = v(x_{m+1}, y_m) = -(2A_m dx + B_m) \quad ,$$

and repeat the process. Note that now we have a new point (x_{m+1}, y_{m+1}) on the free boundary, and the value $v(x_{m+1}, y_{m+1})$.

Since the initial values x_o, y_o, $v(x_o, y_o) = 0$ are known, we can then obtain a sequence of points on the free boundary: (x_1, y_1), (x_2, y_2), etc. The accuracy of our approximations, as can be noted,

improves when dx is taken smaller (or equivalently, when n is large).
The process will stop when v is larger than u_o, for then (6) is
undefined.

Comments. The above should be viewed as a first step towards developin
a predictive algorithm for the velocity and pressure distributions in
the jet flow. A requirement we have imposed on our model is that
it be sufficiently simple and flexible to allow additional physical
considerations (wall friction, etc.).

A Gould plot shows good correlation with the hodographic method.
Relative merits of our scheme over Finite Elements, or other approx-
imating schemes, remains to be investigated.

Birkhoff, G. and Zarantonello, E., Jets, Wakes, and Cavities.
Applied Mathematics and Mechanics 2, Academic Press (1957), pp37-38,
Gallanger, R. H., Oden, J. T., and Taylor, C., Finite elements in Fluids -
Volume I. John Wiley & Sons (1975), pp. 257-279.